Description

Geometrikum

https://geometrikum.jku.at/

Most of the exhibits were made by students and employees of the institute.

March 2, 2026

Geometrikum

Bascetta Star **5**

Desargues's theorem **6**

Dragon curve **7**

Modular origami dodecahedron **8**

Folding paper models) **9**

Gömböc **10**

Hilbert curve **11**

Hyperbolic plane **12**

Hyperbolic paraboloid **13**

Hyperboloid **14**

Great stellated dodecahedron **15**

Truncated Icosahedron **16**

Klein bottle **17**

Lorenz Manifold **18**

Menger-Sponge **19**

Noperthedron **20**

Möbius strip **21**

Oloid **22**

Penrose P3 Tiling 23

Pluecker Conoid 24

Pythagoras tree 25

Rectangles in an icosahedron 26

Rhombenicosidodecahedron 27

Rhombicuboctahedron 28

Rhombicuboctahedron out of acrylic glass 29

Schwarz lantern 30

Scutoid 31

Seifert Surface of Boromean Rings 32

Sierpinski-Tetrahedron 33

Soma Cube 34

Stanford Bunny 35

Stereographic Projection: Visualized using a 3D model 36

The stellated octahedron 37

Tesseract 38

Tensegrity 39

Trefoil Knot 40

Triangulated objects 41

24-cell 42

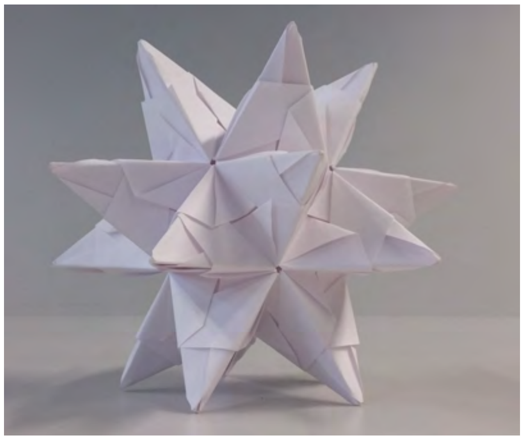
Viviani's Window 43

Voronoi Mosaic 44

Bascetta Star

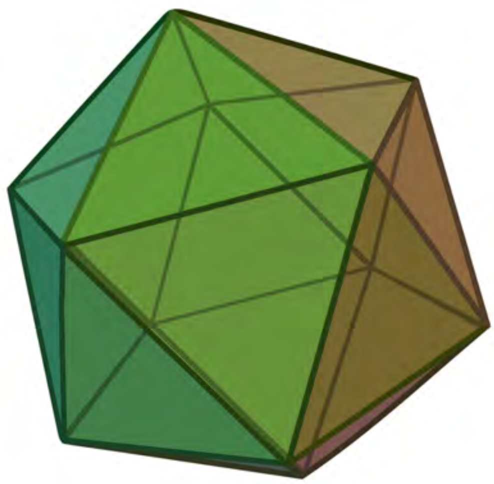
Description

The Bascetta Star is a particularly impressive display of origami art that exhibits a high level of internal stability while requiring neither scissors nor glue to assemble. The trick lies in correctly fitting together a set of 30 paper squares, all of which must be the same size and folded in the same way. The inventor is Paolo Bascetta, an Italian maths teacher.



Important properties

Geometrically speaking one can describe the Bascetta Star as an icosahedron star, which itself is an icosahedron - a 20 sided polyhedron - with a pyramid on each of its faces. The Bascetta Star's structure as a piece of origami art is made up of 30 identical base shapes that are first created by folding square pieces of paper. We call these base shapes tornillo modules. Tornillo comes from the Spanish word for screw, an allusion to the way these base shapes are twisted into each other during assembly, and module comes from how they can all be thought of as individual parts. Impressively, these tornillo modules are not only used in the construction of icosahedron stars. As a matter of fact, all stars based on Platonic or Archimedean solids can be built using these very tornillo modules. The number of tornillo modules needed for assembling each of these stars will always be equal to the number of edges the star's underlying polyhedron has.



DIY guide

`www.mathematische-basteleien.de/bascetta_stern.htm`

Desargues's theorem

Description

Desargues's theorem, named after the French mathematician Gérard Desargues, is, together with Pappus' theorem, one of the closure theorems that form a fundamental basis for both affine and projective geometry as axioms.

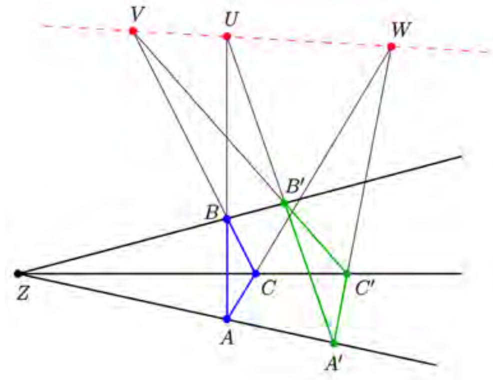
Projective Form: If the lines connecting the vertices of the two triangles $A \leftrightarrow A'$, $B \leftrightarrow B'$ and $C \leftrightarrow C'$, meet at a point Z, then the intersections of the extended corresponding sides U, V, W lie on a single line a.

Conversely, if U, V, W lie on a common line, then the lines connecting the corresponding vertices meet at a point Z.

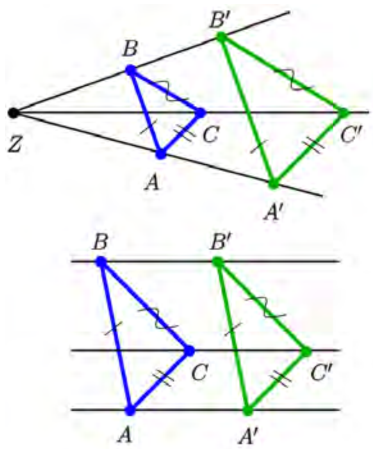
Affine Form: If the lines connecting the vertices of the two triangles meet at a point Z and two pairs of corresponding sides are parallel, then the third pair of corresponding sides is also parallel.
The affine version arises when, instead of a common intersection point Z, the lines AA', BB', and CC' are assumed to be parallel.

Properties

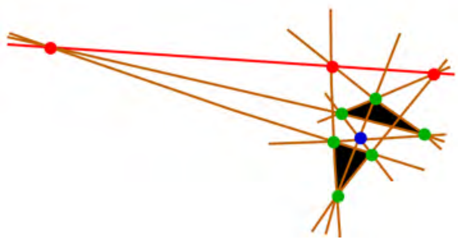
The model visualizes Desargues's theorem in 3D. The two-dimensional configurations can be seen as projections of the 3D structure for example, from the ground floor in front of the skylights.



Projektiver Satz von Desargues
(©https://commons.wikimedia.org/wiki/User:Ag2gaeh)



Affine Versionen
(©https://commons.wikimedia.org/wiki/User:Ag2gaeh)



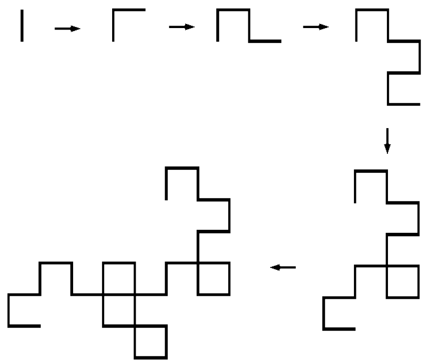
Dragon curve

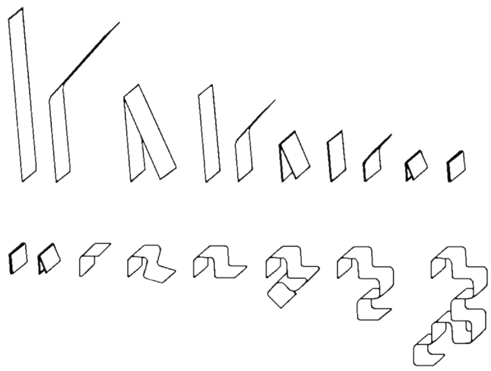
Description

The dragon curve is an object which is, just like the Koch snowflake, constructed iteratively. Drawing the curve happens just like in turtle graphics: „R“ means a 90° turn to the right and „L“ a turn to the left. It starts by drawing an upward line. After this every line is drawn in the current direction after each symbol in the list.

The starting list consists only of the symbol „R“. In each following iteration the preceding list is copied, an „R“ as added and again a copy of the old list is appended, in which the symbols of the list are in reversed order and „R“ and „L“ are swapped.

An illustrative possibility for finding the n-th iteration of the curve is to fold a strip of paper in half n times. If you now open the paperstrip and every side of it is positioned in an angle of 90° you get the n-th iteration of the dragon curve.



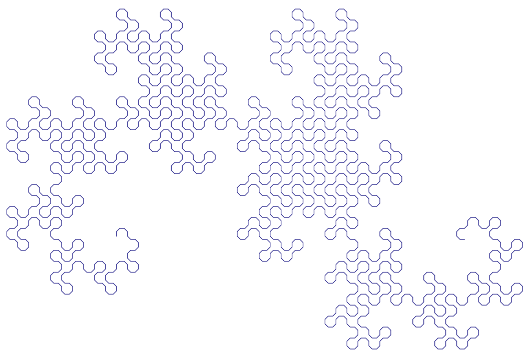


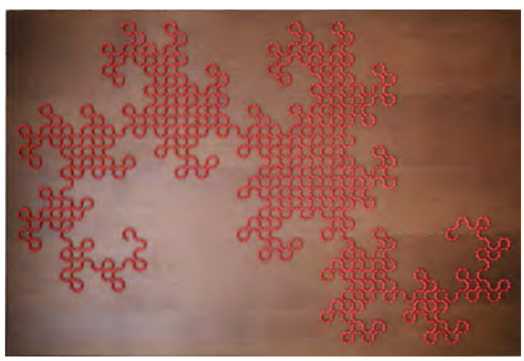
Important properties

The dragon curve is just like the Koch snowflace a fractal curve which is self similar. Even the curve uses a finite area its border is infinitely long.

Although the curve looks rather irregular it never crosses itself even once. Further the dragencurve has a surprisingly simple aspect ration of 3:2.

Also it is an object with which it is possible to tile the plane completely.



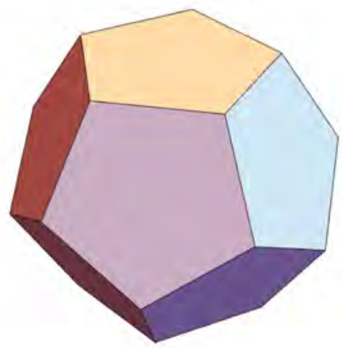


Institute of Applied Geometry

Geometrikum

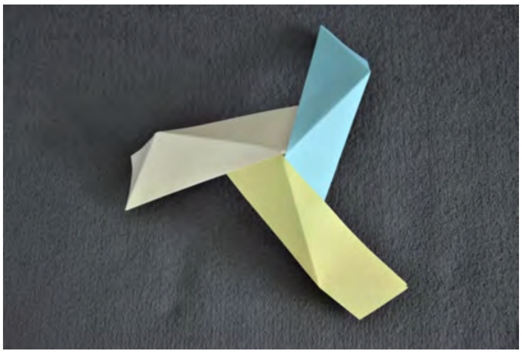
JOHANNES KEPLER UNIVERSITÄT LINZ

Modular origami dodecahedron



Description

A dodecahedron is a 12-sided polyhedron. Dodecahedra come in many different shapes. One of the most familiar shapes of dodecahedra is the regular dodecahedron, which has twelve regular pentagons as faces and is one of the Platonic solids. It has 30 edges, 20 vertices and 160 diagonals. The surface area and the volume of a regular dodecahedron are both related to the golden ratio. The dual polyhedron is the regular icosahedron. With these two polyhedrons there can be constructed many other solids, e.g. the truncated icosahedron (soccer ball), which is the result of taking the intersection of a regular dodecahedron and a regular icosahedron. In total there exists 6 384 634 topologically distinct convex dodecahedra. The number of vertices of these dodecahedra varies from 8 to 20.



Origami model

Modular origami is a paper-folding technique which uses two or more sheets of paper to create large and complex structures.
The modular origami dodecahedron is assembled from 30 paper modules. Each module represents an edge of the dodecahedron.
One module needs six folding steps. Each end of the module consists of a flap and a pocket, which were created during the folding process. By inserting flaps into pockets three modules can be combined to get one vertex of the dodecahedron. Because of this, the modules keep together without the usage of a glue.

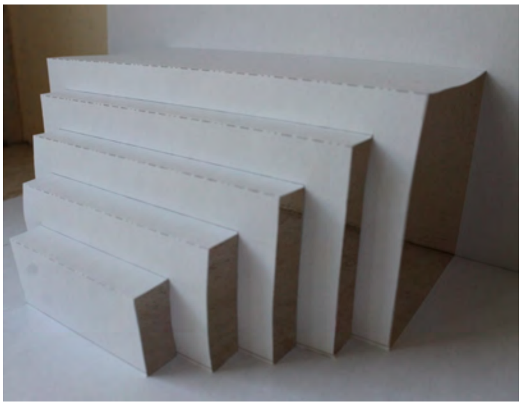
Folding paper models

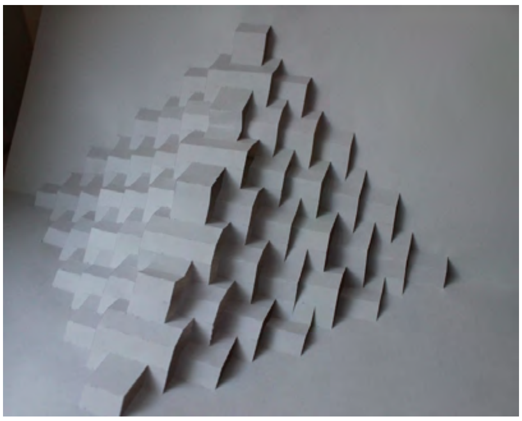
Description

Besides papercutting and paper planes, paper can be used to create a variety of art objects, for example so-called folding paper models. Folding paper models are created by cutting and scoring of paper, which then can be folded into the desired shape. There are no limits to the design options - from simple geometric figures to more sophisticated objects.

Important properties

To create folding paper models only a sheet of paper is needed. A utility knife (commonly known as Stanley knife) qualifies for cutting, whereby a ruler can be used in order to support precise cutting results. Also, a cutting plotter can be used to cut the paper exactly. Furthermore, choosing the right paper thickness is crucial: With a sheet that is too thin, unfolding becomes more difficult, which is why a thicker paper should be used. The colouring, however, depends on one's preferences.

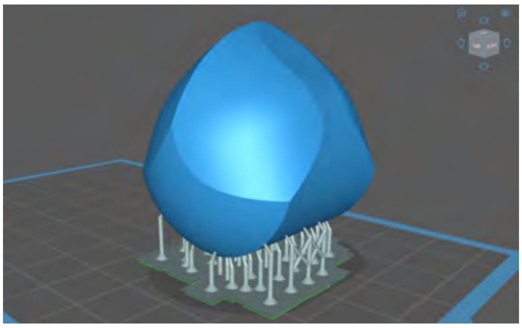


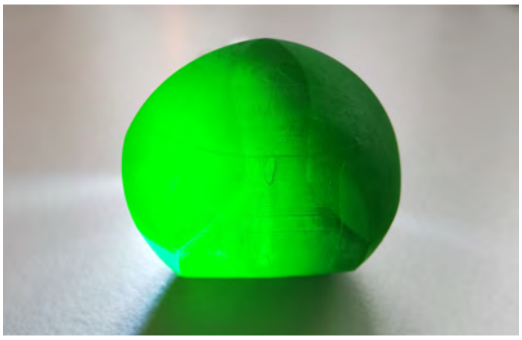


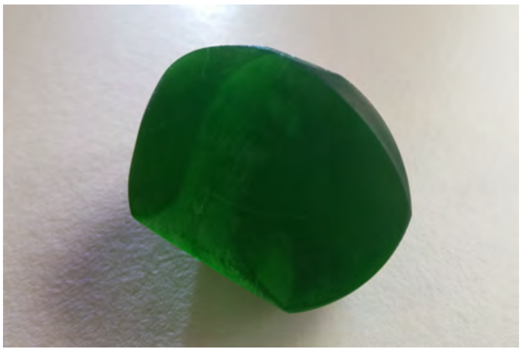
Gömböc

Description

The Gömböc is a body with the special property of being able to roll back to a certain point, no matter how it is placed on the ground. It can be compared to the classic „tumbler toy“, which also gets up again and again from any position. However, this only manages to stand up due to heavy weights inside the body. The two mathematicians Peter Várkonyi und Gábor Domokos were the first to answer the question of the existence of such a convex body without hidden weights with yes and to prove it.



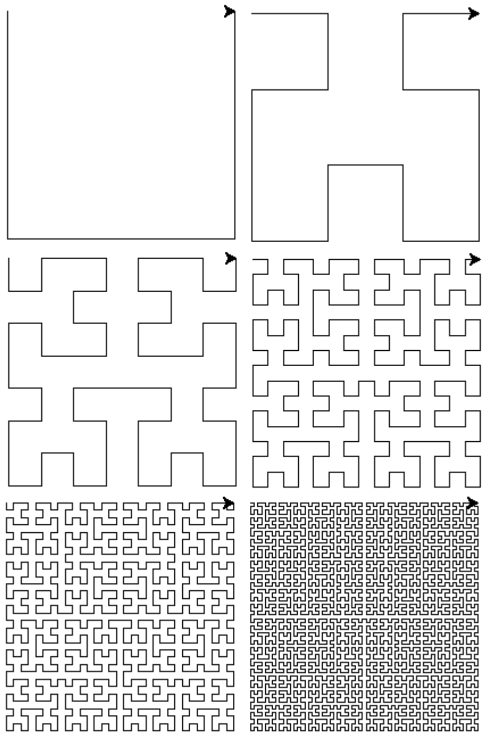




Important properties

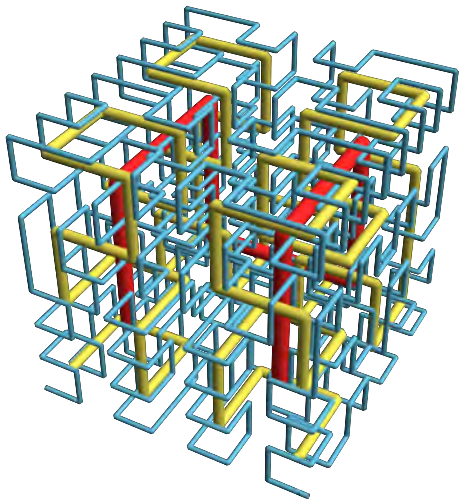
Two outstanding properties of the Gömböc's are the convexity and homogeneity of the mass. One can distinguish between three types of equilibria, the stable, the unstable and the indifferent. In a stable equilibrium, the body returns to the previous position after a disturbance, in an unstable equilibrium, the body will move further and further away from the previous position after the disruption, and in an indifferent equilibrium, the body will take up a new equilibrium position afterwards. The Gömböc has exactly two equilibria, one stable and one unstable. This is remarkable because it was shown that no body exists which has less than two equilibria.

Hilbert Curve



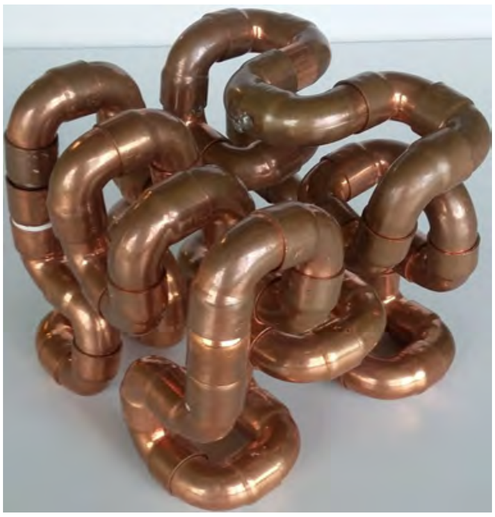
Description

The Hilbert curve, discovered by David Hilbert in 1891, is a continuous curve in mathematics that completely fills the area of a square as a limit curve of polygonal lines. As a FASS curve (space-**f**illing, self-**a**voiding, **s**imple and self-**s**imilar), it can cover a two-dimensional area with a continuous one-dimensional line. This curve, which is unique except for reflections and rotations, is the only two-dimensional FASS curve for the square that starts and ends at two corners. The Hilbert curve therefore offers the possibility of covering an entire two-dimensional region with a single continuous line.



Important properties

The Hilbert curve features a recursive iteration construction. At each step, each segment is divided into 2^d congruent subsegments (d-dimensional intervals) with half the side length, while on the input side an interval is divided into 2^d subintervals of equal length. In this way, the method converts 1D nestings of intervals into 2D nestings of squares (or into 3D nestings of cubes) while preserving inclusion. In 3-dimensional space, significantly more diverse construction possibilities open up, which are characterised by various ”traversals”.



Hyperbolic plane

Description

The hyperbolic plane belongs to the models of surface geometry. This plane has a special property: If you move away from a point in the hyperbolic plane, then the space exponentially expands around this point.
One of the first models of a hyperbolic plane was made by William Thurston in 1970. His model was a paper model and therefore it was very unstable. In 1977, the mathematician Daina Taimina revised Thurstons's model. Her goal was to make it possible to feel the properties of this unique geometry. Thus she created the first crocheted model. Crocheted hyperbolic models usually consist of double crochets, single crochets, chain stitches and slip stitches. They are crocheted in rounds and every n-th stitch will be doubled. The smaller n is, the more crimped the hyperbolic plane becomes.

Important properties

The hyperbolic plane is defined as the 2-dimensional hyperbolic space $\mathbb{H}^2$. An important property of the hyperbolic plane is, that the intersecting curvature is constantly -1. For comparison, the Euclidean space has a curvature of 0 and the sphere a curvature of 1. In the hyperbolic plane, a pair consisting of lines can be parallel, hyperparallel or intersected.

Hyperbolic paraboloid

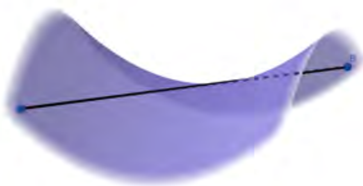
Description

In mathematics, a second-order algebraic surface is called a paraboloid. This means that the coordinates x and y occur at most to the 2nd power. Such surfaces are also referred to as „quadric “. A hyperbolic paraboloid is based on the equation $a^2 \cdot x^2 - b^2 \cdot y^2 = z$. This is different from a hyperboloid. In the case of $a^2 \cdot x^2 + b^2 \cdot y^2 = z$ one would speak of an elliptic paraboloid. In architecture, the hyperbolic paraboloid is often used for roof structures. The first related patent was registered in 1928 by the Russian engineer Tatjana M. Markova.

Important properties

Because of its characteristic shape, the hyperbolic paraboloid is often referred to as the saddle surface because it is oppositely curved in the direction of the two major axes.

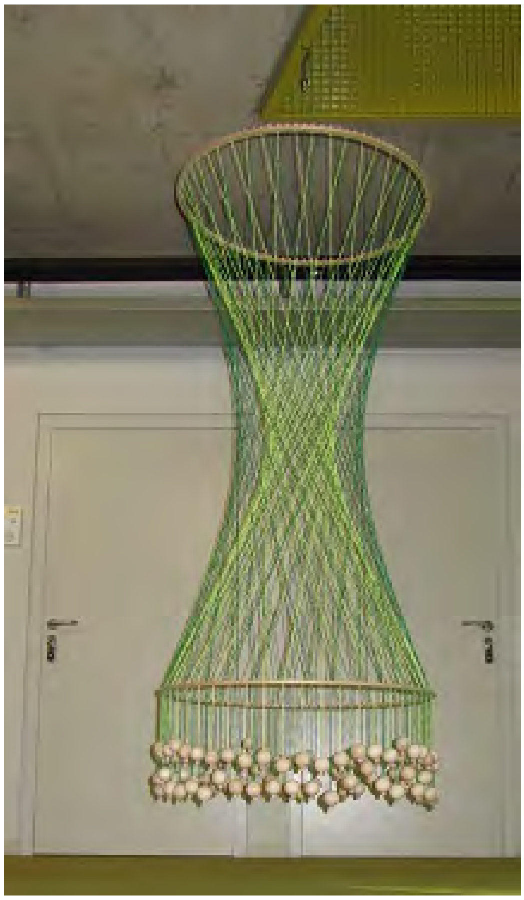
If one cuts the hyperbolic paraboloid in the direction of these two major axes with planes, one obtains in each case a parabola, hence comes the name, which means in Greek as much as „showing a parabola “. When the algebraic surface is intersected with a plane of the third/last spatial direction, we obtain a hyperbola. A special case can be seen in the last picture on the right side, here we see a straight line as intersection between the hyperbolic paraboloid and a plane.



Hyperboloid

Description

A „hyperboloid of revolution“ is a surface created when you rotate a hyperbola around one of the major axes. For example, a hyperboloid is described by the following equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$. There are two forms, the one- and the two-shell hyperboloid (second and third illustration on the right). The first is also interesting for architects and building engineers, a well known example being the cooling tower of a nuclear power plant.



Important properties

Furthermore, the one- or two-shell hyperboloid can merge by changing the parameters a, b, c. The one-shell hyperboloid is made by setting the right side of the equation to $+1$, which is also called hyperbolic hyperboloid. The two-shell hyperboloid is obtained by setting the right side to -1, which is referred to as the elliptical hyperboloid. This transition is also called „hyperbolid of revolution “. If you cut the hyperboloid with a horizontal plane, you get a circle. If you cut it with a vertical plane, you get a hyperbola. As can be clearly seen from the object, the one-shell hyperboloid can be formed of straight lines.

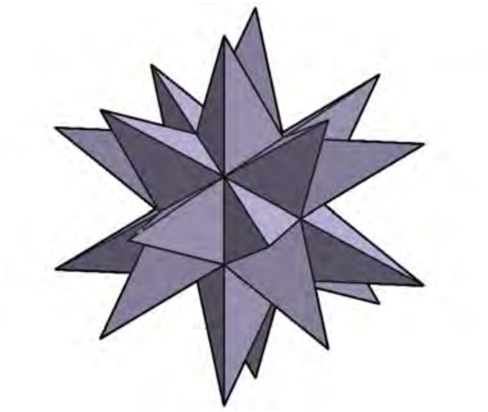
Institute of Applied Geometry

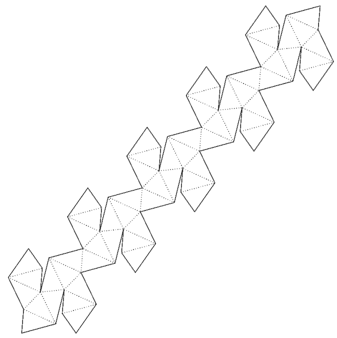
Geometrikum

Great stellated dodecahedron

Description

Together with the small stellated dodecahedron and the great dodecahedron as well as the great icosahedron the great stellated dodecahedron represents one of four Kepler-Poinsot polyhedra. As a regular geometric object it is bounded by 12 uniform pentagrams which generate 60 isosceles triangles and it is characterized by equality of both any interior and exterior angles with a value of 63,44°.





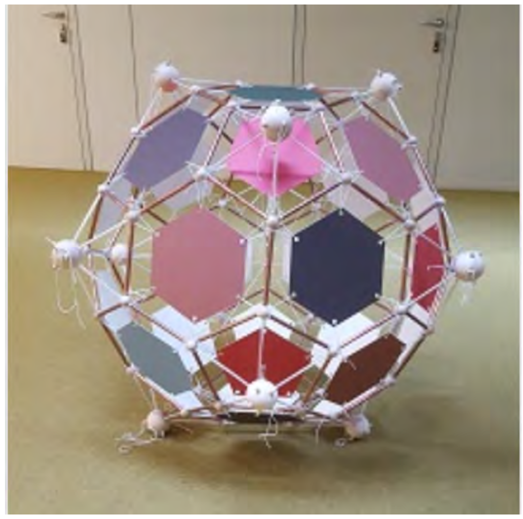
Important properties

The construction of a great stellated dodecahedron can be performed by extending all edges of an icosahedron beyond the corners until always three of them intersect each other in a common point. Therefore, the great stellated dodecahedron can be perceived as an icosahedron with 20 pyramids attached to it whose tips each correspond the spikes of the star-shaped polyhedron itself as well as the corner points of a regular dodecahedron with equal surface area but greater void volume. Alternatively, the great stellated dodecahedron can also be regarded as geometry circumscribed by 12 mutually intersecting pentagrams which are coincident with the pentagonal cut surfaces of the icosahedron.
The adjacent figures illustrate the geometry and net of a great stellated dodecahedron as well as the via a miniature photovoltaic system illuminable transparent paper model of such a polyhedron in switched-on and -off state, manufactured within Geometrikum by Hannah Maria Weiss in winter semester 2022/23.

Truncated Icosahedron

Description

The truncated icosahedron is one of the five platonic solids, more precisely the truncated icosahedron is a polyhedron and belongs to the archimedean solids. The word icosahedron is derived from the Greek $\epsilon\iota\kappa\emptyset\sigma\acute{\alpha}\epsilon\delta\rho\emptyset\emptyset\nu$ eikosáedron and means twenty-faced. An icosahedron consists of 20 equilateral triangles, these triangles form together 12 corners. The truncated icosahedron is formed by truncation of the corners of an icosahedron. If all edges of the truncated icosahedron have the same length, it is called a regular truncated icosahedron. The shape of the regular truncated icosahedron is strongly reminiscent of a football. Therefore the regular truncated icosahedron is often referred to a football solid. Another example of a truncated icosahedron is the fullerene C_{60}.



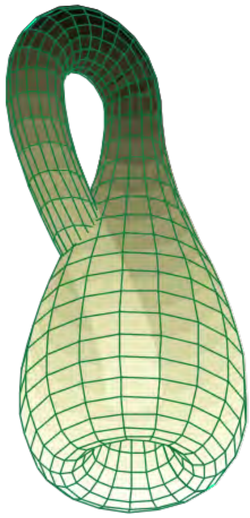
Important properties

As mentioned above, an icosahedron consists of 20 equilateral triangles, which form 12 corners. By blunting the 12 corners you get 12 regular pentagons and the original 20 equilateral triangles become regular hexagons. Therefore the truncated icosahedron consists of 32 surfaces, 60 corners and 90 edges. The regular truncated icosahedron has also 90 sides, but these are of equal length. The solid dual to the truncated icosahedron is the pentakis dodecahedron.

Klein bottle

Description

The Klein bottle, which was named after the mathematician Felix Klein, is a geometric object that consists of only one side. This means that one cannot distinguish between the inside and the outside. It is therefore possible to pass from the supposed inside to the outside without crossing an edge. In this respect, a Klein bottle is comparable to the Möbius strip, which has only one side and one edge.
The construction of a Klein bottle is possible with the help of a square, which is folded into a tube. The ends are then connected so that the tube self-intersects. Another possibility is to join two mirrored Möbius strips. This method was used for the crochet Klein bottle shown on the right. Here the Möbius tapes are connected with the sewn zipper.



(©https://commons.wikimedia.org/wiki/User:Tttrung)

Important properties

Klein's bottle has no volume and, unlike the Möbius strip, no edge. Another difference to the Möbius strip is that it cannot be embedded in three-dimensional space. The three-dimensional Klein bottle is just a way to illustrate this object. If the Klein bottle is embedded in the four-dimensional space, the misleading self-intersection can be avoided.

Lorenz Manifold

Description

Weather forecasting is an inaccurate science. Especially the patterns of clouds and their effects are a mystery. One of the first scientists who ventured into the mysteries of weather was the meteorologist Edward Lorenz. In 1963 Lorenz worked on a calculation for weather forecasting, more precisely he developed a simplified mathematical model for atmospheric convection. From these calculations he received the Lorenz Attractor, from which the Lorenz Manifold was developed in 2002. In 2002 the mathematician Hinke Osinga discovered that the computer model of Lorenz Manifold looks like a crochet script. Osinga did not hesitate for long and crocheted the first Lorenz Manifold. Her model consists of 47 rows and 21.989 stitches. The model of the geometricum consists like the original model of 47 rows and 21.989 stitches. In total more than 1 km of yarn was used to crochet the model.

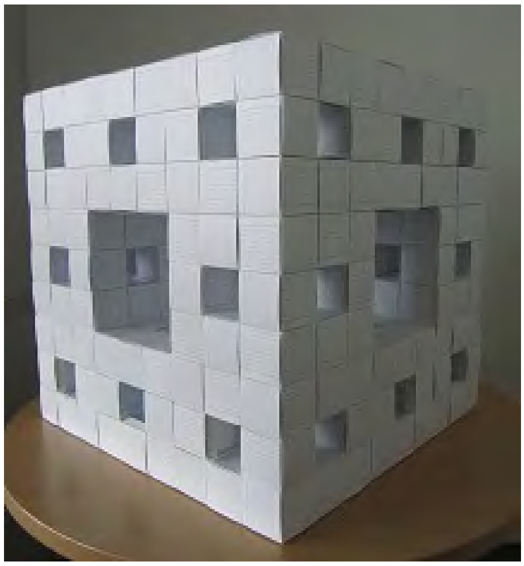
Important properties

The Lorenz Attractor is an infinitely long trajectory in three-dimensional space that never cuts itself. Osinga and Krauskopf looked at the Lorenz system in a new time independent way and thus obtained the Lorenz Manifold. They defined the Lorenz manifold as the surface of all points in space that don't result in the convection role.

Institute of Applied Geometry

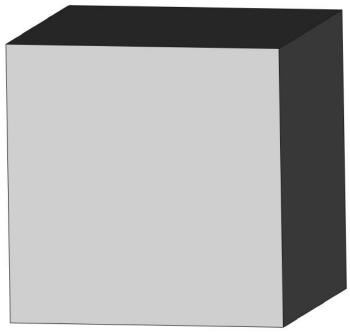
Geometrikum

Menger-Sponge



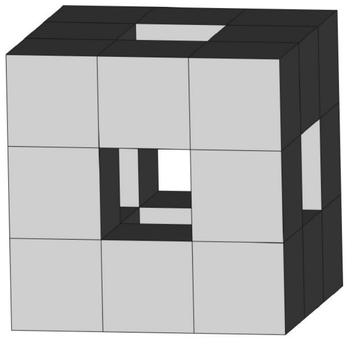
Description

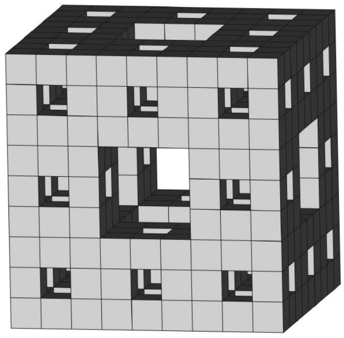
The Menge-Sponge was first described in 1926 by Karl Menger in his studies about the concept of topological dimension. The sponge belongs to the family of fractals, therefore it consists of several smaller copies of itself. Such a Menger-Sponge can be constructed iteratively. A two-dimensional generalization of the sponge is possible as well, in this case it is called Sierpinski-Carpet.



Important properties

In order to create a Menger sponge iteratively, one will start with a cube which is partitioned into $27 = 3 * 3 * 3$ smaller cubes. The next step is to remove 7 of those cubes: precisely the small cubes in the center of each face and the one in the very center of the original large cube. If these steps are repeated infinitely many times, this process leads to an undermining of the cube. Overall the surface converges to infinity, meanwhile the volume goes to zero. The dimension of the Menger sponge can not be given as a natural number, instead it has a crooked (fractal) dimension, which lies between the two-dimensional surface and a three-dimensional cube. The exacte value is constituted by $dim = \frac{\log(20)}{\log(3)} = 2.726833..$.
The picture in the top right corner shows a model of the Menger-Sponge which is one of many sponges displayed at the Institute of Applied Geometry (JKU). This model in particular is made from 400 small paper cubes.





Noperthedron

Description

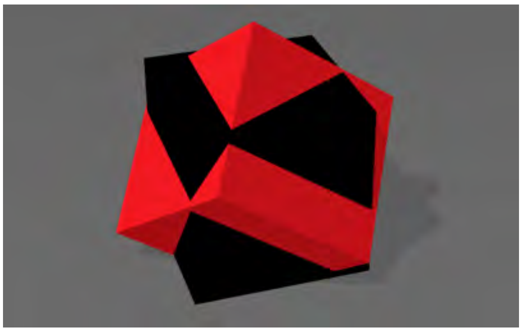
The model consists of two parts. The object on the right shows the Noperthedron, the first polyhedron which does not satisfy the “Rupert's Property”. It was discovered by two Austrian mathematicians in mid-2025.
The 3D printed model (picture 2) shows on the left side a simple cube placed.
The 3rd picture shows a cube going through itself, clearly satisfying the Ruperts's property and the 4th picture shows its projection.

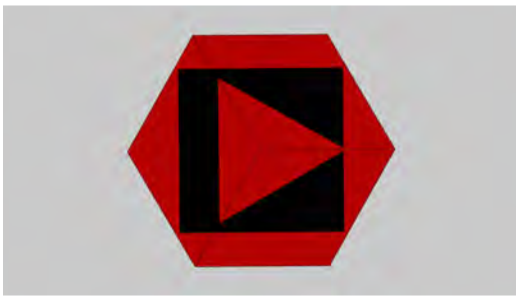
Important properties

The Noperthedron is the first known polyhedron that does not posess Ruperts property, this means that it is not possible to cut a hole into it through which an exact copy of the Noperthedron can pass through.
In contrast, the cube does possess this property. As illustrated by the model, when the cube is oriented appropriately (lying face down) it can pass entirely through its own shadow and thus through itself.
The name derives from Prince Rupert of the Rhine, a 17th-century nobleman and scientist who conjectured that a cube could pass through a hole made in another cube of the same size.

Rendered model

3D-printed model


Rupert's property for a cube


Shadow projection of a cube going through itself

Möbius strip

Description

A Möbius strip (also Möbius band or Möbius loop) denotes a surface with only one edge and one side. It can be easily formed by attaching the ends of a paper strip together with a half-twist. Independent of each other this object was first mathematically described in 1858 by Johann Benedict Listing and August Ferdinand Möbius.
The Möbius strip is not orientable, this means that it is impossible to distinguish between clockwise and counterclockwise turns with consistency. In detail, an arrow pointing clockwise on the strip would return as an arrow pointing counterclockwise.

Important properties

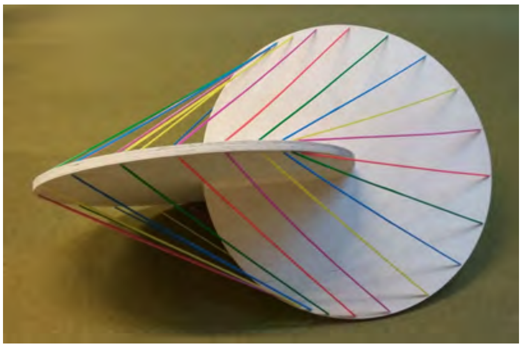
The Möbius strip also appears in hidden form in many other non-orientable surfaces. If the inside and outside of a surface can no longer be distinguished, then there exists a closed Möbius strip on this surface.
For the centerline of a Möbius strip it is impossible to form a circle, expect the strip gets stretched in a certain area. If a Möbius strip gets cut along the centerline, this produces one single strip with two half-twists (no Möbius strip anymore). Cutting the strip a third of the way across its width instead yields two linked stripes (paradromic rings). The result is one new Möbius strip formed from the middle part and one with two half-twists.

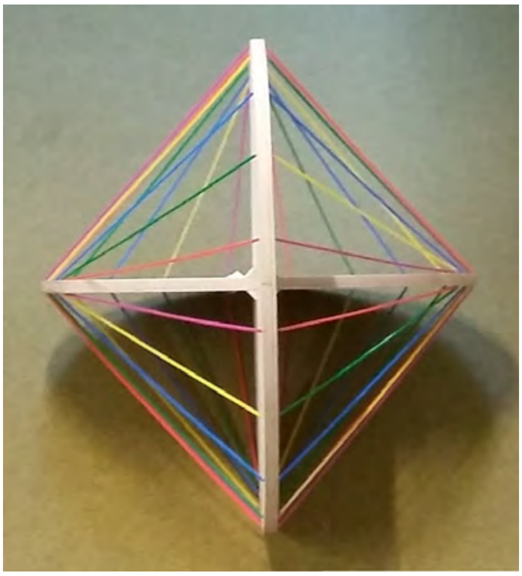
Link to the applied modular design

Oloid

Description

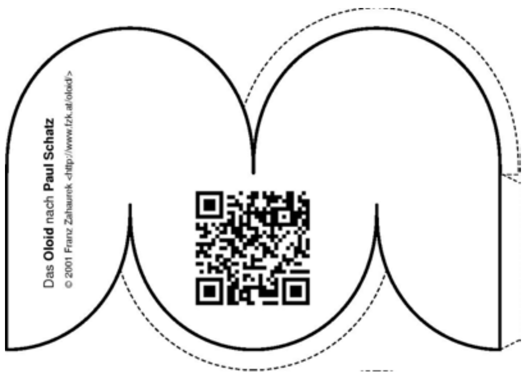
The oloid is a geometric shape with extraordinary properties. It was discovered in 1929 by Paul Schatz together with the invertible cube. It has no corners and only two edges. It is constructed as the convex hull of two circles with the same radius, which are perpendicular to each other. The center of one circle lies on the other circle (see first image on the right side). When the oloid rolls down an inclined surface, every point on its surface touches the inclined surface at some point in time. This means the surface is a developable surface and can easily be represented as a net in the plane, meaning it can easily be crafted from a sheet of paper (see third image on the right side).

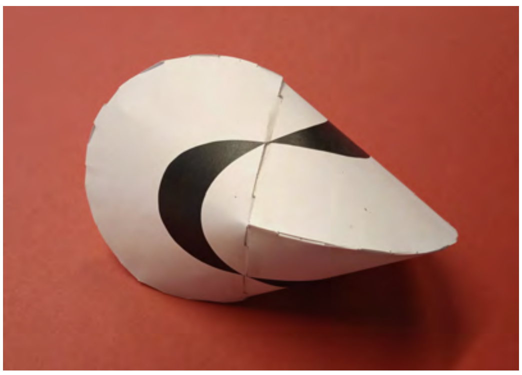




Important properties

If one observes the diagonals of the invertible cube during a complete rotation, these describe the surface of the oloid. Furthermore, the oloid is used in technology as an agitator for stirring, circulating, or aerating. Moreover, the surface area of the oloid is exactly the same as that of the sphere with the same radius as the generating circles.





JOHANNES KEPLER UNIVERSITÄT LINZ

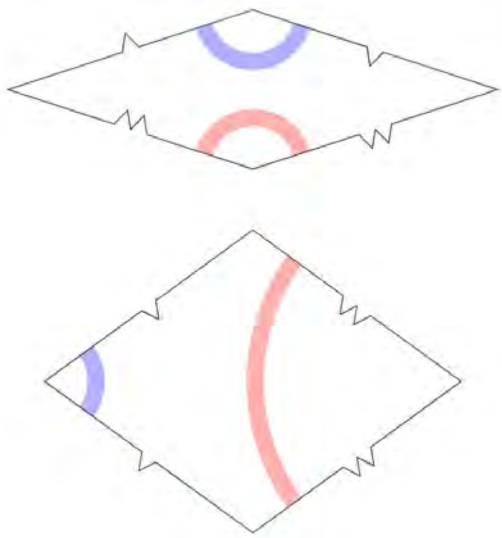
Penrose P3 Tiling

Description

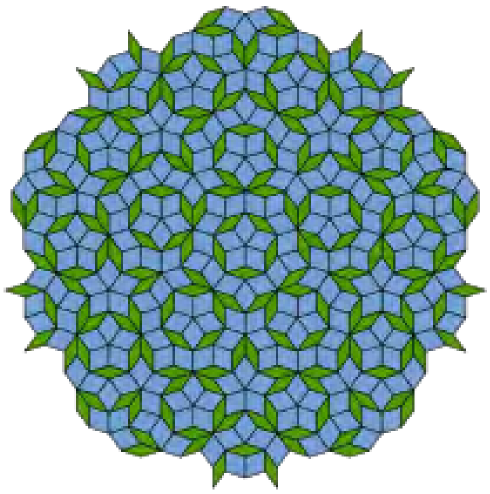
The Penrose Tilings are a family of aperiodic tilings discovered in 1973 by the British mathematician and physicist Roger Penrose. One of these tilings is the Penrose P3 tiling, which consists of a thick and a thin rhombus with matching rules indicated by circular arcs and edge modifications.
A tiling is a covering of the plane by non-overlapping polygons, and a tiling is periodic if it has two translational symmetries. This means we can move a periodic tiling along two different directions, and the two resulting tilings will look identical to the original tiling. An example would be a grid tiling. On the other hand, a tiling is said to be aperiodic if it is not periodic.

Important properties

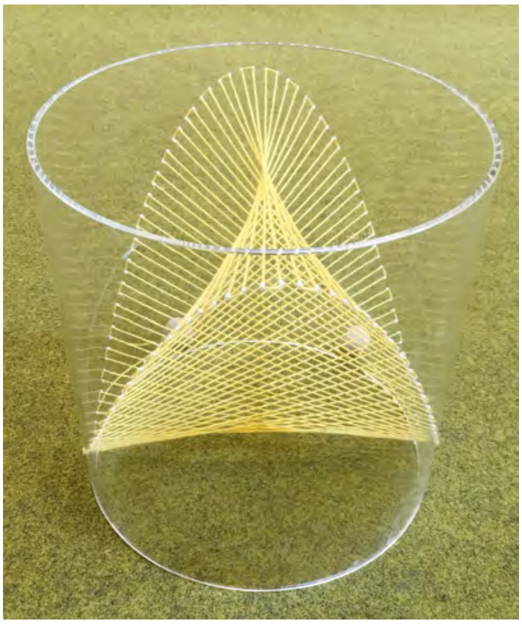
Although Penrose Tilings are aperiodic, meaning they lack translational symmetries, they can exhibit 5-fold rotational symmetry. In fact, aperiodic tilings are closely related to the study of quasicrystals (solid matter with non-periodic structure often exhibiting rotational symmetry, e.g. Aluminum-Manganese (Al-Mn) Alloy).
The golden ratio $\Phi = \frac{1+\sqrt{5}}{2}$ plays a central role in the construction of the P3 tiles. If the sides of the rhombs have length 1, we have that the longer diagonal of the thick rhomb has length Φ and the shorter diagonal of the thin rhomb has length $\frac{1}{\Phi}$.



made by wiki/User:Hagman see
(© https://commons.wikimedia.org/wiki/File:
Penrose-tiles-bump-and-color-coded.svg



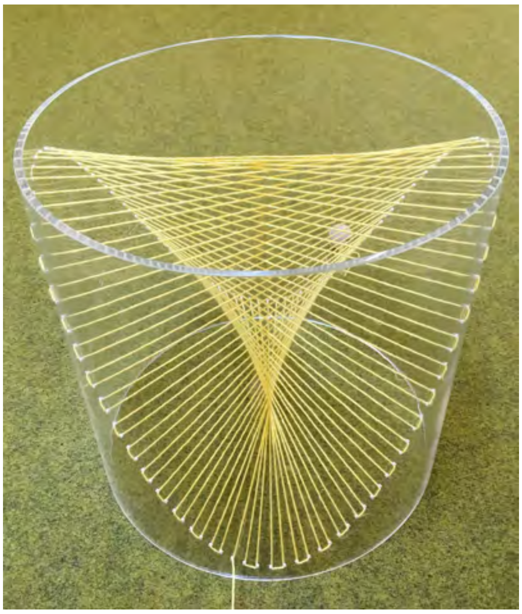
Plücker Conoid



Description

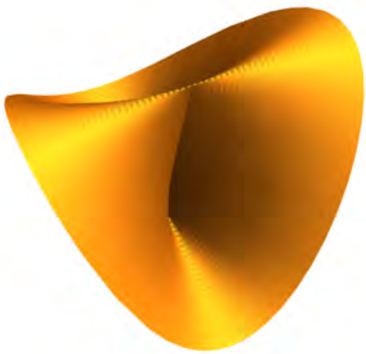
Plücker‘s conoid, named after the German mathematician Julius Plücker, is a *ruled surface*. These surfaces have the property, that there exists a straight line in the surface for every point of the conoid. Such a line is called a *generating line*.
A conoid satisfies two further conditions:

- There exists a socalled "*Richtebene*" that is parallel to all the generating lines.
- There exists a line that is intersected by every generating line. This line is called *axis*.



Plücker‘s conoid can be constructed by rotating a horizontal line around the z-axis with an oscillatory motion and periodicity 2π along the z-axis. Furthermore, it is a right conoid, since the rotation-axis is horizontal to the directrix, which is parallel to the x-y-plane. Plücker‘s conoid is also called conical wedge or cylindroid.

Important properties



The implicit characterization of Plücker‘s conoid is fulfilled for the whole z-axis. But all points on the z-axis are singular. This implies that there exist no tangential planes. Conoids are of high interest for architects, because they can be built using bars. Especially right conoids can be manufactured very easily.

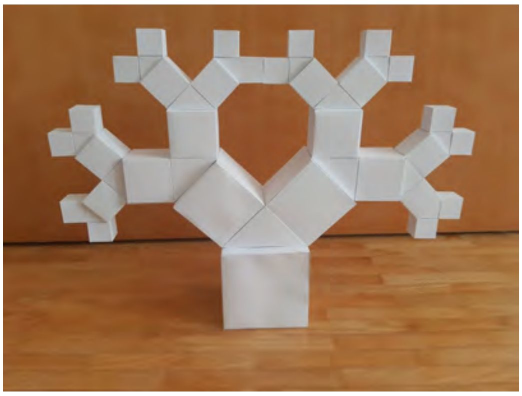
Pythagoras tree

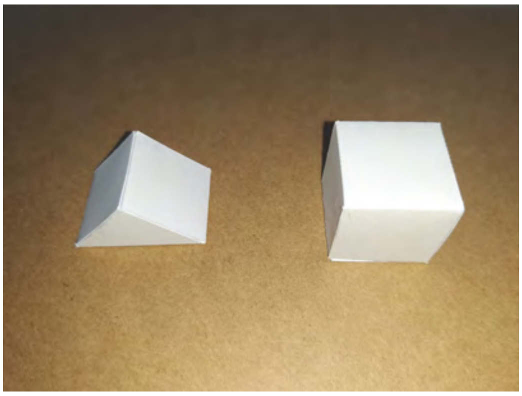
Description

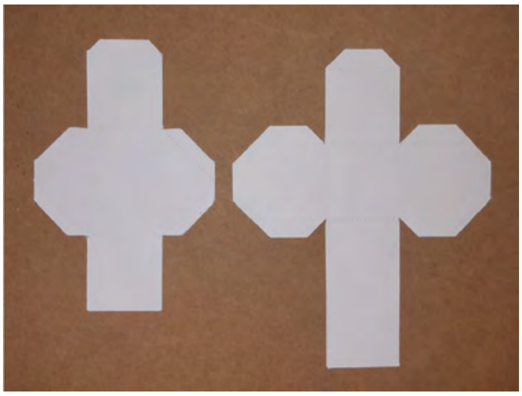
The Pythagoras tree is a recursively created fractal, which was initially described in 1942 by the Dutch maths teacher Albert E. Bosman. The construction is based on the Pythagorean theorem: Starting with a square representing the trunk, two smaller squares are placed on it confining a right angle. This proceeding is recursively applied to the subsequent squares. After some iterations a construct, which resembles a tree, is obtained.

Important properties

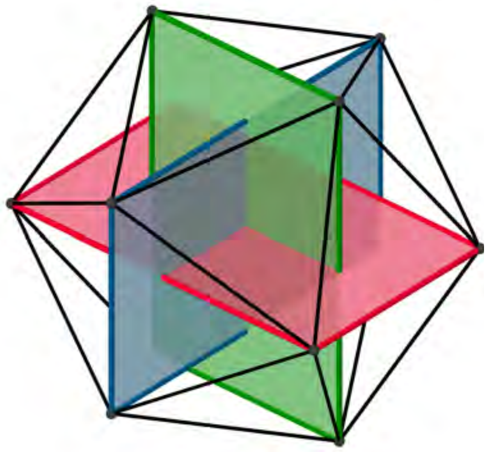
In the case of a symmetric Pythagoras tree the two upper squares confine the lower one with an angle of $45°$ and hence, surround an isosceles triangle. If the length of the trunk is a, then the squares in the i^{th} step have a length of $(\frac{1}{\sqrt{2}})^i \cdot a$. The height of the tree converges to $4 \cdot a$ and the width converges to $6 \cdot a$. Of course, the triangles confining by the sqaures do not have to be isosceles. In this case, a asymmetric tree is formed and the height and width are again bounded.
The two pictures show a three dimensional variation of a symmetric Pythagoras tree. For this, cubes of length $(\frac{1}{\sqrt{2}})^i \cdot a$ instead of squares were arranged.





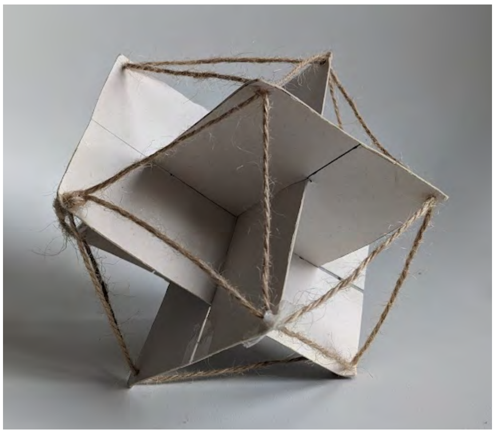


Rectangles in an icosahedron



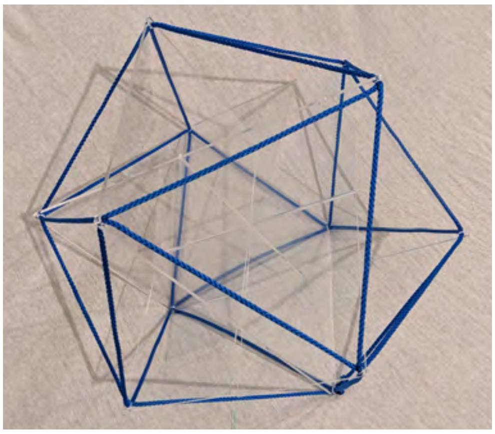
Description

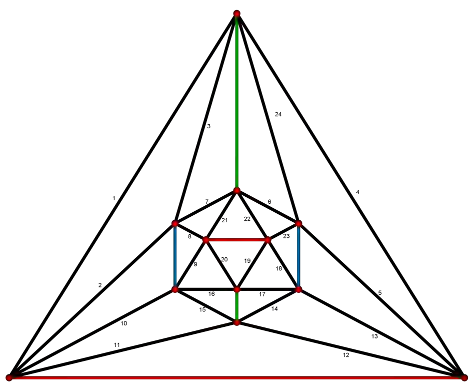
The model shows an icosahedron which is spanned by three golden rectangles. A rectangle is called *golden rectangle* if it is composed of a square and a rectangle, such that both rectangles are similar. The aspect ratio of the rectangel is the golden ratio.
If we now combine three of these rectangles such that, they have the same center point then we get by connecting neighboring points a Platonic solid with 20 sides.



Important properties

An icosahedron has 12 corners, 20 equilateral triangles as lateral surfaces and 30 edges of the same length. The 12 corners are the same points as the corners of the three rectangles. If we depict the icosahedron in a Schlegel diagram we get the adjecent picture, in which one can see, that all corners have an even degree when disregarding the coloured lines. Thereby an euclidean cycle exists which means that all edges can be traced without moving along one edge twice. One possibility for such an euclidean cycle is displayed in the adjecent diagram and was used to connect the corners when constructing the model.

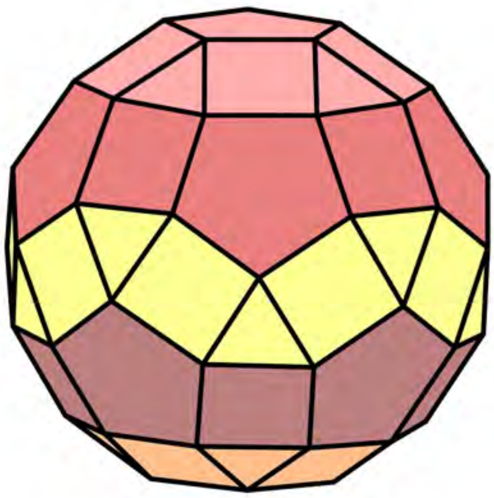




Rhombenicosidodecahedron

Description

The rhombenicosidodecahedron is one of a total of 13 (without similarity difference, otherwise 15) Archimedean solids which were probably discovered in the 3rd century BC by the mathematician Archimedes. The rhombicosidodecahedron gets its name from the congruence of the squares with the rhombuses of a rhombic triacontahedron. In total, the solid consists of 20 equilateral triangles, 30 squares, 12 regular pentagons and 120 edges, with 10 edges each forming a regular decagon. With the help of the outer sphere and tangent planes, the deltoid hexacontahedron results as a dual body.



Important properties

To construct the polyhedron, the corners of an icosahedron, alternatively those of the dual body, are truncated. At the edges of the icosahedron the squares are created and with 5 triangles meeting each other the pentagons.
All Archimedean solids satisfy the formula $(2\pi - \sigma) * E = 4\pi$ where σ describes the sum of all angles and E the number of corners. All polyhedra are convex bodies, thus the validity of Euler's polyhedron formula is given. In addition, all Archimedean solids have the uniformity of the corners.
The shown rhombenicosidodecahedron was made in the course of the geometry seminar using plywood plates.

Rhombicuboctahedron

Description

The rhombicuboctahedron consists of 18 squares and 8 equilateral triangles. Therefore it has 48 edges and 24 vertices. Centred at the origin and the 11 permutations of the coordinates (± 1, ± 1, $\pm(1 + \sqrt{2})$) for the vertices the rhombicuboctahedron has edge length 2. The polyhedron, as can be seen in figure 1, can be dissected into two square cupolae (top and bottom) and an octagonal prism in the middle. The name originates from the fact that 12 of the 18 squares are congruent to the rhombi of the rhombic dodecahedron.

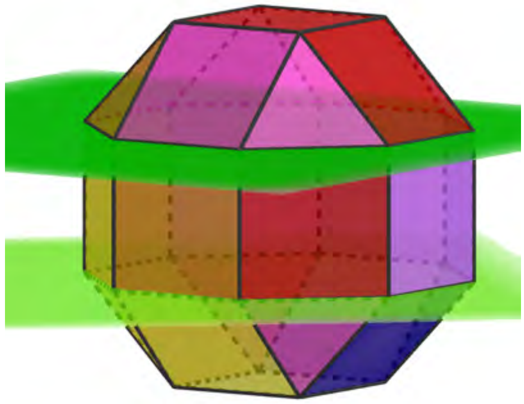


Abbildung 4: Rhombenkuboktaeder

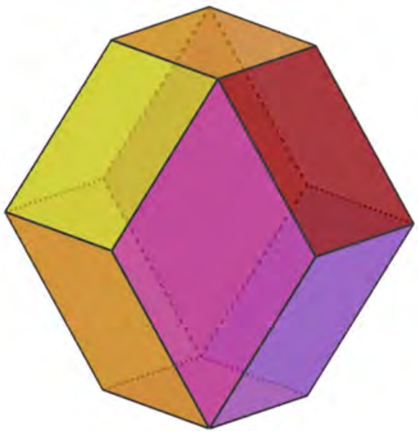


Abbildung 5: Rhombendodekaeder

Abbildung 6: Modell

Important properties

The uniform polyhedron is neither a Platonic solid nor a prism nor an antiprism. Combined with the property of the identical vertices it therefore is one of the Archimedean solids. The pseudo rhombicuboctahedron is a result of either of the two square cupolae turned, so that the top and bottom equilateral triangles are not opposite each other, but shifted. The polyhedron can be obtained by cutting off the edges of a cube or octahedron, so that the cut areas are square. Each triangle borders four squares and each square either four squares or two squares and two rectangles.

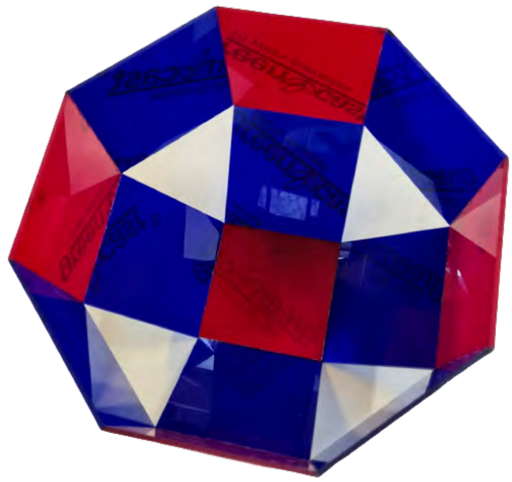
The polyhedron is also utilized in the uniform space-filling tessellations.

Rhombicuboctahedron out of acrylic glass

Description

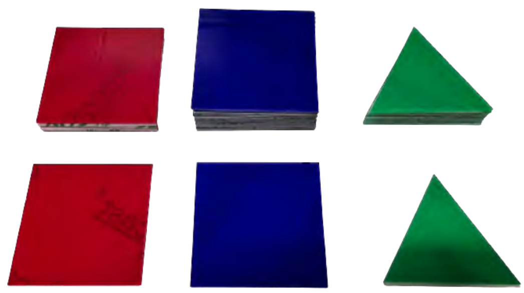
The rhombicuboctahedron is a convex polyhedron. It is made of 18 squares and eight equilateral triangles, where the sides of the squares and triangles are of equal length.
In this object the side lengths of the squares are 118 mm. Therefore, the total height of the rhombicuboctahedron is 285 mm.
The plane angle between two squares is 135° and between square and triangle it is 144.7°. Therefore, before assembling the object, the edges of the individual acrylic glasses were bevelled to fit. Thus, the colours of the respective planes come into play. The eight triangles are green. Those six squares that only adjoin squares are red and the remaining 12 squares are blue.
When building the object, first the “inner ring” of eight squares – alternating blue and red – was glued together. Then the remaining squares were placed above and below the ring. Finally, the triangles were attached. Special plastic glue was used as adhesive.

made by en.wiki User Cyp using POV-Ray, see
(© https://en.wikipedia.org/wiki/User:Cyp/Poly.pov



Important properties

The rhombicuboctahedron belongs to the Archimedean solids. It has 48 edges and 24 vertices.
The dual of the rhombicuboctahedron is the deltoidal icositetrahedron.



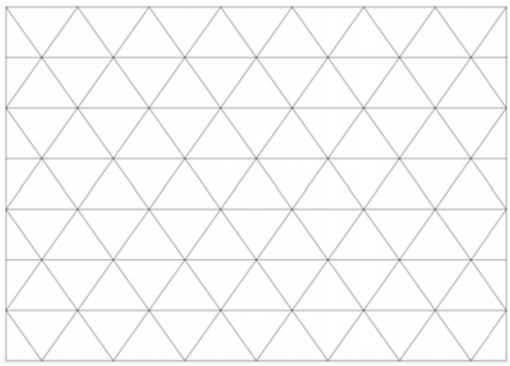
Schwarz lantern

Description

Named after the mathematician Hermann Amandus Schwarz "Schwarz lantern" is created by triangulation of a circular cylinder. A cylinder is split into k equally high disks with the height $\frac{H}{k}$, where H denotes the total height of the cylinder. The circular area of each slice is approximated by a regular, n-edge with the regular polygons twisted to each other. The corner points of the n-corner are connected by triangles.
The "Schwarz lantern" can be modeled by certain folding constructions out of a paper.

Important properties

Special about the "Schwarz lantern" is its shell surface. Depending on the choice of parameter for n and k, it converges to the surface of the cylinder, or assumes any arbitrary size.
For $n \to \infty$ and $n \to \infty$ as well as $k \to \infty$, where k grows slower than n^2, the surface of the "Schwarz lantern" clings to the cylinder. On the contrary, the lateral surface diverges to infinity for $k \to \infty$. The "Schwarz lantern" belongs to the group of bodies with finite volume, but infinite sheaths.



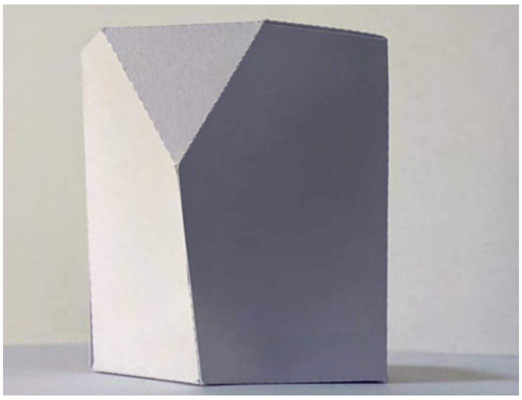
Scutoid

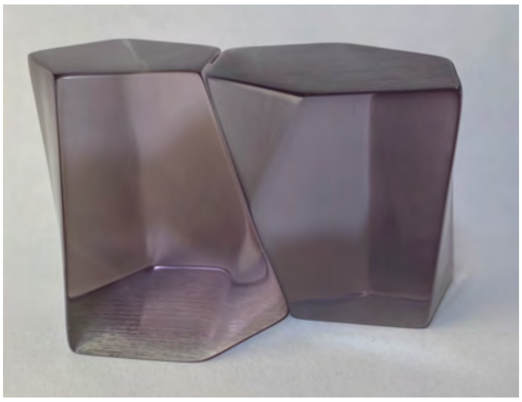
Description

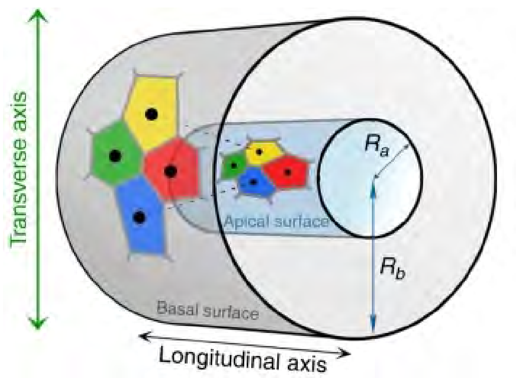
Scutoids are composed of two base planes that have different numbers of edges and at least one further vertex, that is not part of any base plane. Due to that their side surfaces are in general bent. Yet it is especially interesting, that one can construct Scutoids that pack tightly together (meaning that when putting two together the shared side surfaces fit one another perfectly and avoid any space between them). Another special property of Scutoids is, that the neighbouring relationship on the base planes changes due to that extra vertex when going from one plane to the other.

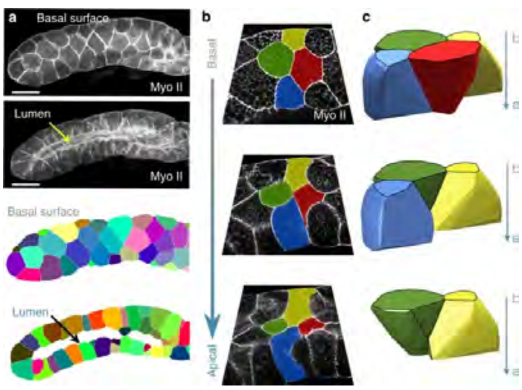
Importance in nature

Scutoids have just recently in 2018 been discovered by a Spanish research group consisting of cell biologists and mathematicians during their attempt on modelling cell structures. They were investigating how cells behave in bend epithelial tissue and concluded that cells can change their neighbours. But this was not explainable by models using prisms or prismatoids as cell forms. Only Scutoids made it possible to model this behaviour and the authors therefore concluded that they are essential for bent epithelial tissue to form. In fact, the authors even found scutoid-like cells in nature.









Source of graphics:
Gómez-Gálvez, P., Vicente-Munuera, P. et al.
Scutoids are a geometrical solution to three-dimensional packing of epithelia.
Nat Commun 9, 2960 (2018).
https://doi.org/10.1038/s41467-018-05376-1

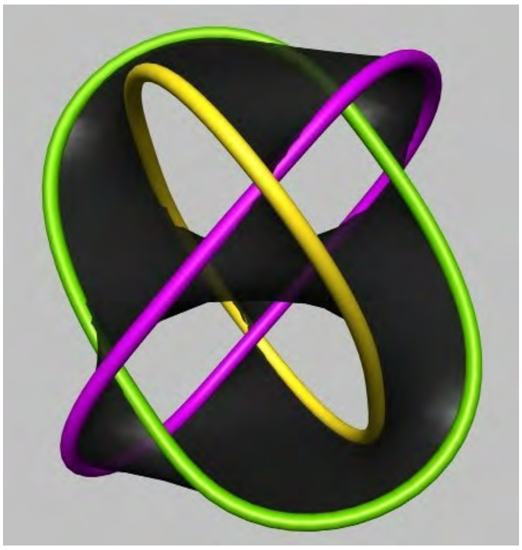
Seifert Surface of Boromean Rings

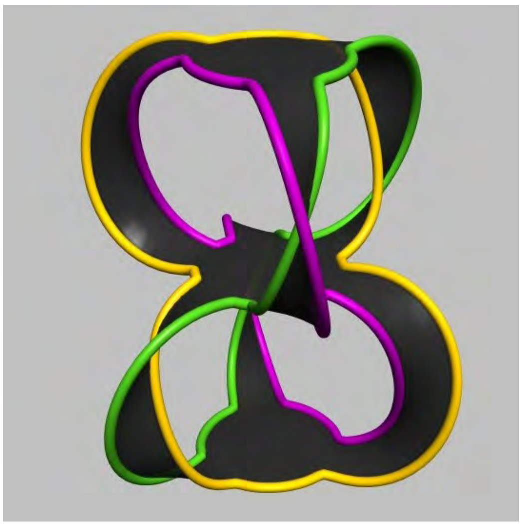
Description

The Boromean rings are a link of three closed curves, which can not be separated unless one of the rings is removed (property of a Brunnian link).
A Seifert surface is an oriented surface, which is bounded by a link or knot. In 1961 Martin Gardner presented the Seifert surface of Boromean rings in his column for the first time.

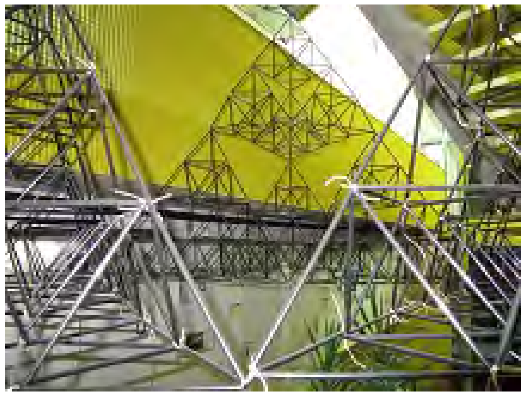
Important properties

Different to the two dimensional space, it is not possible to construct the Boromean rings with circles in the three dimensional space. It can, however, be done with non-circular shapes such as ellipses.
The Boromean rings are an example for an alternating link, which means that the crossings alternate between going over and under.
In 1934 Herbert Seifert puplished a proof, that any link has an associated Seifert surface. For this he used an algorithm, now named after him, which computes the surface of any given knot.
The Seifert surface of Boromean rings can also be represented by three discs which are connected by twisted bonds (this was the blueprint for the crochet pattern of the model).



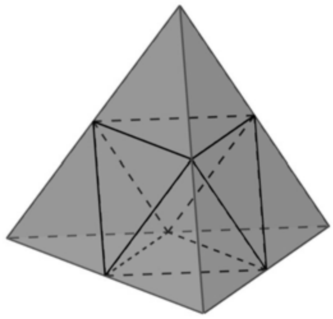


Sierpinski-Tetrahedron



Description

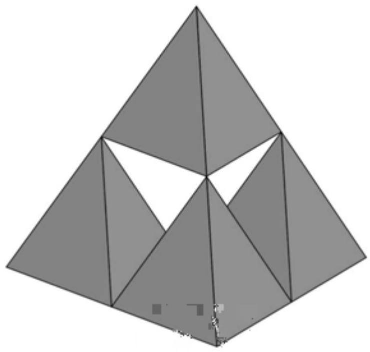
The polish mathematician Maclaw Sierpinski was the first to describe the Sierpinski-Triangle in the year 1915. It belongs to the family of fractals, therefore it is self-similar to an (usually equilateral) triangle across different scales. One can create such a figure iteratively. Also a three-dimensional representation of the Sierpinski-Triangle is possible, instead of a triangle, a tetahedron is used as starting figure.

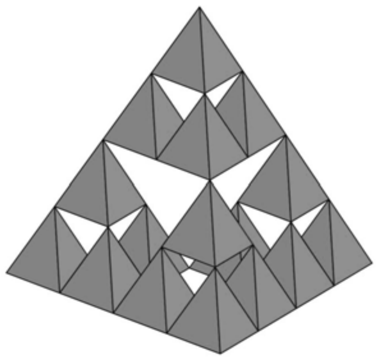


Important properties

For an iterative creation of a Sierpinski-Tetrahedron a octahedron with half the edge length of the original tetrahedron is cut from the center of the very same. Obviously there are four smaller tetrahedrons remaining, during the next step another octahedron is removed from each of them. If this process is repeated infinitely many times, the most important characteristics of the Sierpinski-Tetrahedron become visible: the volume converges to zero, whereas the length of the edges goes to infinity and the surface stays constant. Although the figure is in $3D$, the dimension is given by $dim = \frac{\log(4)}{\log(2)} = 2$.

The picure in the top right corner shows a model of the Sierpinski-Tetrahedron, which is displayed at the Institute of Applied Geometry (JKU). It was made from straws during the „Long Night of Research“ (2014).





The soma cube

Description

The Soma cube was invented in 1936 by Piet Hein during a lecture on quantum mechanics conducted by Werner Heisenberg. Its primary function is as an dissection puzzle with the goal of constructing a 3x3x3 cube from seven smaller, individual parts. All these pieces are built differently but from the same smaller, identical cubes. The intended benefit for the player is improved spatial imagination and ability to solve problems.

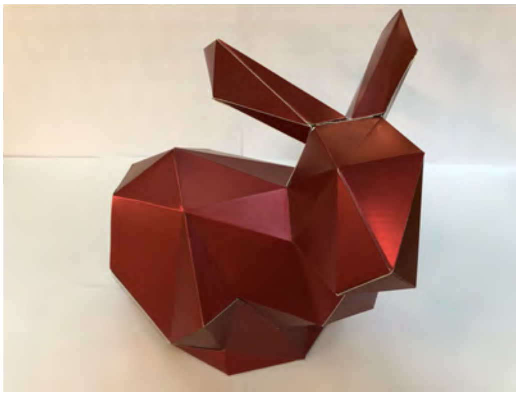
Important properties

In 1961 it was calculated that there are exactly 240 ways to solve the base game, except for when the cube is rotated or flipped. In each of these solutions there is only one place that the **T** piece can be placed. As mentioned above, the pieces of the cube consist of smaller, identical cubes (also called polycubes) - each polycube is constructed with at least three smaller cubes, but no more than 4. Therefore, there is one of order three and six of order four. Within the six of order four pieces, there are three flat (L-, S- and T-shaped) and three 3D forms.

Stanford Bunny

Description

The Stanford bunny is a computer graphics 3D test model developed by Greg Turk and Marc Levoy in 1994 at Stanford University. The model consists of data describing 69,451 triangles determined by 3D scanning a ceramic figurine of a rabbit. This figurine and others were scanned to test methods of range scanning physical objects. The data can be used to test various graphics algorithms, including polygonal simplification, compression, and surface smoothing.



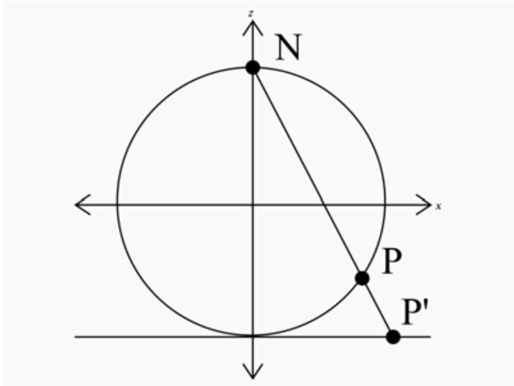
Important properties

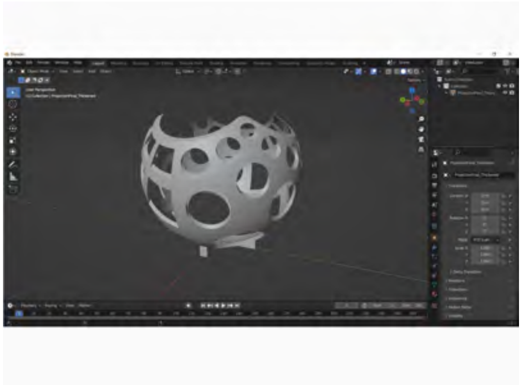
The original model consists out of 69,451 triangles and was created by using multiple scans from different perspectives. However, a much smaller number of triangles is needed to get an accurate to shape replica of the Stanford Bunny. Meanwhile numerous reproductions out of different materials exist and even a paper version is possible, for which a cut-out sheet can be used. In 2016, a model of Lego bricks was constructed at the Institute of Applied Geometry at the JKU.

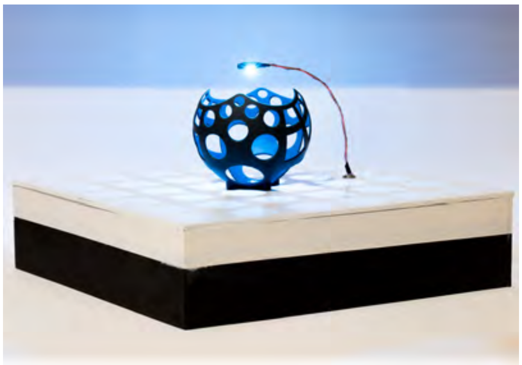
Stereographic Projection: Visualized using a 3D model

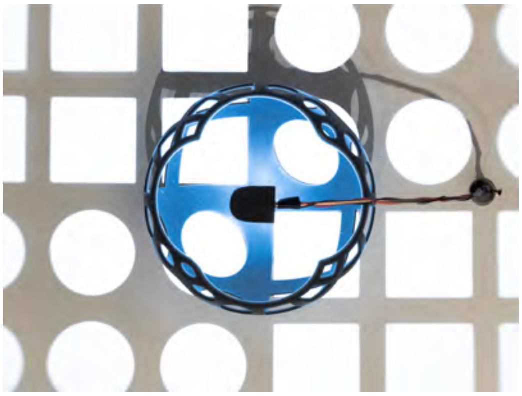
Description

A **stereographic projection** is used for mapping a sphere onto a plane. More specifically, a *perspective* projection, so a projection where all lines originate from one single point, is used. This point is located at the shell of the sphere. Opposite to it, a tangent plane is used as the projection plane.









Important properties

Circles are preserved: Projections of circles are again of circular shape.
Angles are preserved: Angles on the sphere are of the same size as their projected counterparts.
Non-isometry: Distances are not necessarily preserved, which means that other shapes than circles are typically distorted.

The 3D printed model

The model was manufactured using the 3D-printing technology SLA. A LED, installed at the north pole of the sphere, forms the projection center. For visualizing the mentioned properties, some cutouts were added to the sphere. When looking at the shadows, one can clearly see that the circles, as well as the 90° angles are preserved, while the square shaped shadows are coming from heavily distorted shapes (non-isometry).

The stellated octahedron

Description

Paper is ubiquitous. From early childhood on we are used to it, be it as an exercise book or a craft project. Thanks to its versatility it is suitable for a variety of projects. Additionally, paper is cheap, easy to work with and available everywhere.
The stellated octahedron is depicted for the first time by Leonardo da Vinci in Pacioli's book 'De Divina Proportione' in 1509. It gets its name 100 years later in 1609 by Johannes Kepler who calls it 'Stella Octangula'.

Important properties

The stellated octahedron may be viewed as being a polyhedron compound or a stellation of an octahedron. Seen as a compounds, it is the simplest of five regular polyhedral compounds, and the only regular compound of two tetrahedra, built as the union of a tetrahedron and its dual tetrahedron. The eight vertices of the resulting polyhedron are the vertices of a cube. The intersection of the two tetrahedra form an inner octahedron which share the same face-planes with the compound. Seen as a stellation, it is built by adding small tetrahedra on each face of an octahedron.

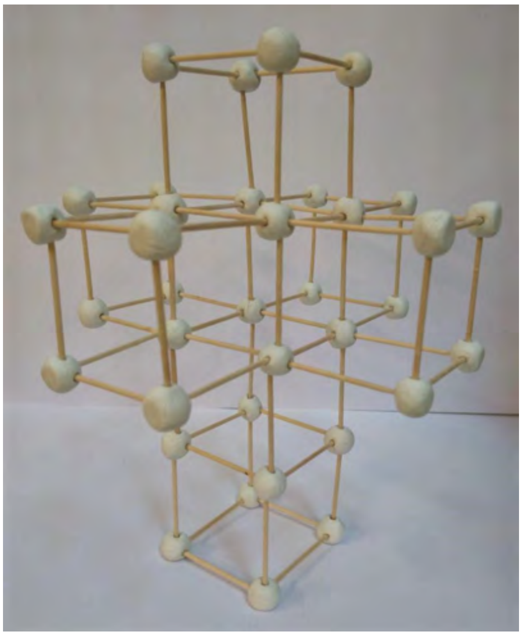
Tesseract

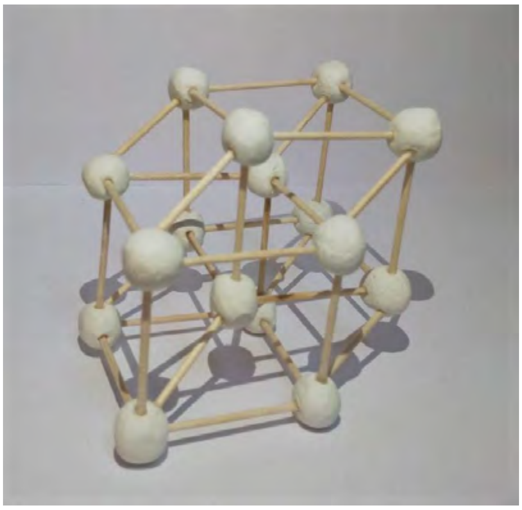
Description

A hypercube is an n-dimensional analogy to a cube or a square. A 4D-hypercube is also referred to as Tesseract. The name is derived from the ancient greek $\tau\epsilon\sigma\sigma\epsilon\rho\epsilon\varsigma\ \alpha\kappa\tau\iota\mu\epsilon\varsigma$ (tésseres aktínes), which can be translated with „four rays“. In order to recieve a 3D-projection of the Tesseract, one will start with it's 3D-net (shown one the first picture). By cutting and folding it can be turned into the second figure, before reaching the final representation. Over all there are 261 possibilities to display the 3D-net of a 4D-Hypercube.

Important properties

The „cube in cube“-representation, which is shown on the bottom right picture, is the only possibility to obtain all important properties of the Tesseract: it has 16 vertices, 32 edges, 24 faces and 8 cells. The volume and surface of a cube with the edge-length a can be computed as follows: $V = 8 * a^3$ and $A = 24 * a^2$.





Tensegrity

Description

The Tensegrity Cube is a vivid illustration of how structures can support themselves and remain in balance with the aid of tensioned chains or ropes alone. The idea for tensegrity structures came from Richard Buckminster Fuller Jr. and Kenneth Snelson, the former also coining the name tensegrity by combining the words tension and integrity. Areas of application for tensegrity include architecture, robotics, anatomy, biochemistry and art.

Important properties

Tensegrity structures consist of compression and tension elements. The compression elements are stable, inflexible parts that give the structure a certain degree of stability. They also serve as anchor points for the tension elements. The tension elements are flexible parts that are constantly under stress, keeping the structure in balance. The compression elements are isolated from each other and only connected by the tension elements. Thanks to their special design, tensegrity structures require fewer materials and can still withstand a great deal of stress.

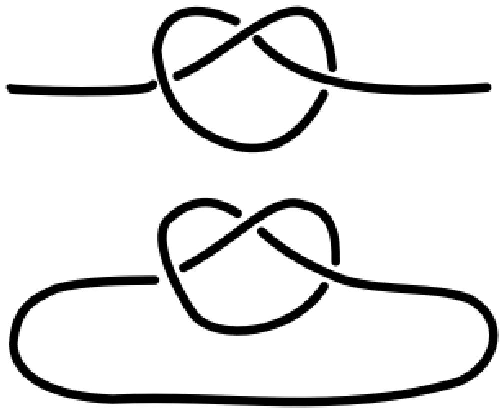
NASA SUPERball Tensegrity Lander Prototype

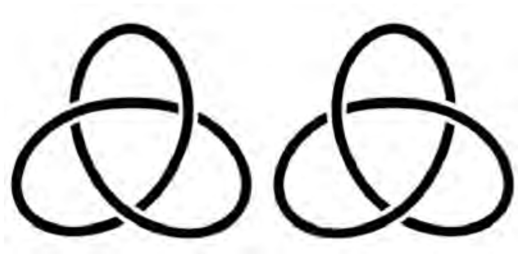
(©https://commons.wikimedia.org/w/index.php?title=User:Sunspiral)

Trefoil Knot

Description

The trefoil knot is the most basic non-trivial knot and is fundamental to the study of mathematical knot theory. A mathematical knot is a closed curve in three dimensions. A trivial knot is a closed loop without any knot tied into it, e.g. a circle in three dimensions. A non-trivial knot is a knot which cannot be untangled, meaning that one can not deform it into a trivial knot. We can for example obtain a trefoil knot by connecting the loose ends of an overhand knot, as seen in the picture on the right.
Furthermore, the trefoil knot is the only knot with crossing number 3, meaning that the minimal amount of crossings in any 2 dimensional diagram of the knot is equal to 3. The trefoil knot is also chiral, meaning that it is different from its mirror image. The two resulting variants are known as the left-handed trefoil and the right-handed trefoil.





Cultural Importance

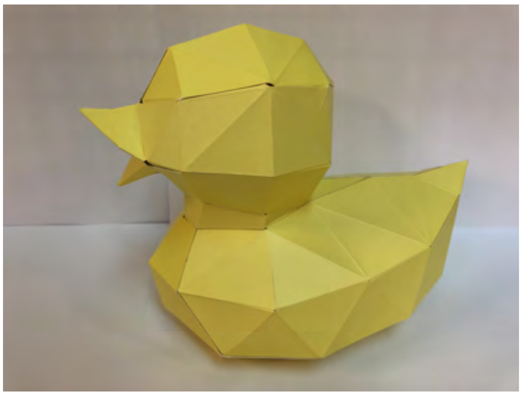
The trefoil knot is a common motif in visual arts. For example, the triquetra symbol (a two-dimensional representation of the trefoil knot) was already used 5000 years ago in Native America. The triquetra is one of the best-known symbols in Celtic and Germanic culture. The picture on the right shows a Celtic Cross with four triquetras.

https://commons.wikimedia.org/wiki/File:Cruz_Celta_con_Trisquetas.svg

Triangulated objects

Description

Many real objects in three-dimensional space do not have an easy-to-imitate structure. Nevertheless, sometimes a (computer) model of such an object is useful, in order to be able to perform further calculations and simulations. For this purpose, the object is approximated by flat surface patches trying to maintain the object's original shape. If all the surface patches are triangles, this type of model is called triangulation.

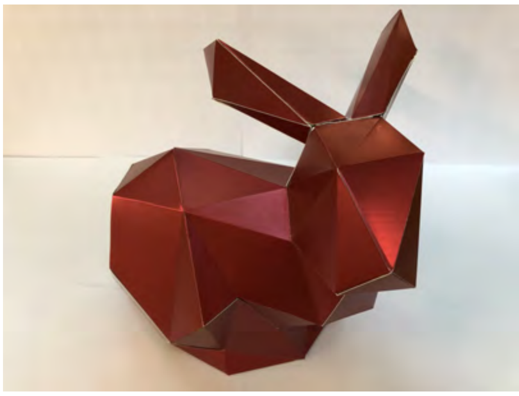


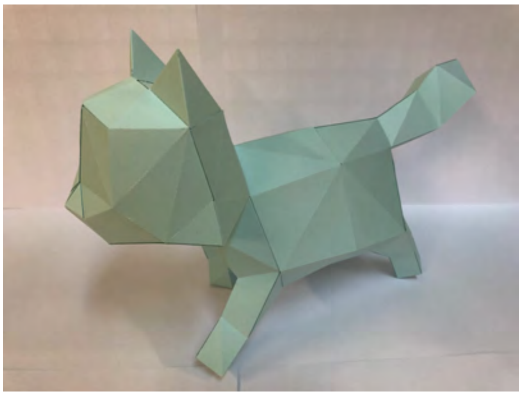
Important properties

In the case that the object resulting from the triangulation is a convex polyhedron (a convex polyhedron is given if the line segment of any two points of the object lies entirely within the polyhedron), Euler's polyhedron formula can be applied. This formula describes the relation of the number of edges (E), vertices (V) and faces (F):

$$E + F = V + 2.$$

This means that the sum of the edges and the faces equals the number of vertices increased by two. This particularly holds for any triangulation of the sphere.





24-cell

Description

The 24-cell is a convex regular four-dimensional polytope, which was first described by the swiss mathematician Ludwig Schläfli in the mid-19th century. It is also known as icositetrachoron, octaplex, icosatetrahedroid, octacube, hyper-diamond or polyoctahedron, being constructed of octahedral cells. Ludwig Schlalfli defined the 4-dimensional relatives of the Platonic bodies: for the tetrahedron the 5-cell (pentachoron), for the hexagon the 6-cell (tesseract), for the octahedron the 24-cell (icositetrachoron), for the dodecahedron the 120-cell (hecatonicosachoron) and for the icosahedron the 600-cellar (hexacosichoron).

Important properties

The edge of the 24-cell consists of 24 (regular) octahedron cells, of which 6 meet at each corner and 3 at each edge. The body contains, in addition to the 24 cells, 96 triangular planes, 96 edges and 24 corners.
The present model was built as part of a seminar at the Institute of Applied Geometry of Metal and Fishing Rope.

Institute of Applied Geometry

Geometrikum

Viviani’s Window

Description

Viviani’s window is a figure eight shaped curve on the sphere, which is named after the Italian mathematician and physicist Vincenzo Viviani (1622-1703) and was described by him in 1692.
The curve is obtained as the intersection of the sphere with a circular cylinder of half the radius, which touches the sphere tangentially at one point (the double point of the curve).

Important properties

Viviani’s window is the intersection of the sphere

$$x^2 + y^2 + z^2 = r^2$$

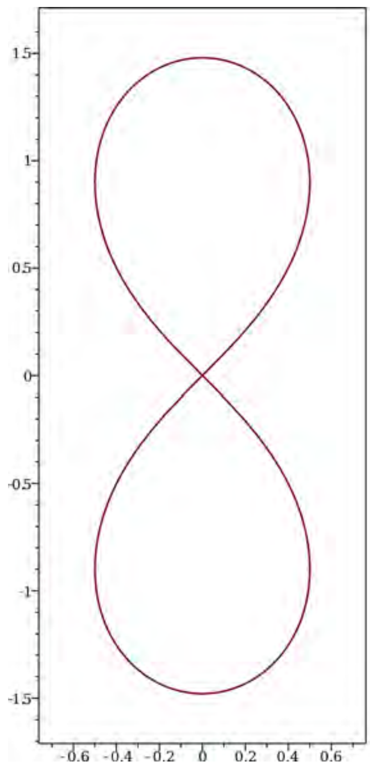
with the cylinder

$$x^2 - rx + y^2 = 0$$

and is therefore an algebraic curve of fourth order. The difference of these equations gives the equation

$$z^2 + rx = r^2$$

of a parabolic cylinder, which also contains the curve. The curve admits a parameterization by rational functions, since it possesses a double point.
In addition, Viviani already established that the area of the sphere without the window is $8r^2$, hence one may construct a square with the same area using only ruler and compass.



Voronoi Mosaic

Description

The Voronoi cell is an important data structure in computational geometry. It was first studied in the n-dimensional case by the Russian mathematician Georgy Voronoi in 1908. In two dimensions, a Voronoi diagram is defined on a set of points in the plane, where each region of the diagram consists exactly of those points that are closer to the corresponding site than to any other point in the set.
Such a region can be interpreted as the intersection of half-planes, which implies that Voronoi cells are convex. This makes it possible to reproduce images relatively well using Voronoi diagrams.

Euclidean Voronoi diagram
(©https://commons.wikimedia.org/wiki/User:Balu.ertl)

Construction

The underlying point set was generated from a digital image and then transformed into a Voronoi diagram. Since the small Voronoi cells would have been difficult to cut in glass, acrylic glass was used instead. For this purpose, the image was first printed and fixed between two acrylic plates using VHB tape. Subsequently, in Inkscape, two paths were created for each Voronoi edge in order to achieve a better 3D effect, which were then engraved using a laser cutter.